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TECHNICAL REPORT
SRL-0122-TR

AN IMPLEMENTATION OF WAVELET ANALYSIS

by

C.J. Coleman

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AN IMPLEMENTATION OF WAVELET ANALYSIS

by

C.J. Coleman

SUMMARY

Discrete wavelet analysis is described through the dilation equation approach. A practical implementation of these techniques is developed and the resulting computer software is included as an appendix.

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1 INTRODUCTION

As with Fourier analysis, the basic idea behind wavelet analysis is to decompose a function into the sum of meaningful 'basis' functions. Unlike Fourier methods, however, the basis functions of wavelet analysis exhibit some form of localisation in both the frequency and time domains. The desirability of such a decomposition is well recognised and has led to the development of the short time Fourier transform. In what follows, it will be shown that wavelet techniques have several features that make them an attractive alternative to the short time transform.

The following report describes some of the basic ideas that underlie a discrete wavelet analysis. A non rigorous approach is used, but there are several references to more complete descriptions. The major purpose is to give the reader sufficient understanding to allow a meaningful application of the software that was developed as part of this project. Appendix A contains some FORTRAN subroutines that are sufficient for a basic wavelet analysis. In addition, Appendix B describes an associated PC software package that can perform a fairly comprehensive analysis. Several examples of its use are included and these amply demonstrate the potential of a wavelet approach.

The report draws heavily on some papers by Daubechies [1] and Mallat [2,3]. In addition, extensive use has been made of an excellent introductory paper by Strang [4] and a recent tutorial paper by Rioul and Vetterli [5]. For more advanced material, the reader should consult the books by Daubechies [6] and Ruskai et al [7].

2 THE SCALING FUNCTION

In Fourier analysis, the basis functions are generated by scaling the argument of a single function ϕ ($\phi(t)=\exp(it)$). If the sequence ϕ_n ($n\in Z$) is generated according to $\phi_n(t)=\phi(nt)$, it forms a basis for $L^2[-\pi,\pi]$ (the space of Lebesgue square integrable functions on $[-\pi,\pi]$). In the case of wavelet analysis, the basis elements are generated from a single function by both a scaling and a translation. Consider the problem of representing a function in terms of elements ϕ_{ij} ($i,j\in Z$) that are generated from the single function ϕ according to $\phi_{ij}=2^{\frac{j}{2}}\phi(2^j x-i)$. The simplest example of ϕ is the box function

$$\phi(x) = \begin{matrix} 1 & 0<x\leq 1 \\ 0 & otherwise \end{matrix} \tag{1}$$

which leads to a step approximation $A_J f$ for the function f where

$$A_J f = \sum_i a_i^J \phi_{iJ} \tag{2}$$

and the value of J sets the level of approximation. As illustrated in Figure 1, the accuracy will increase as J grows (2^{-J} is the width of a box at level J if the boxes have unit width at level J=0).

For the above ϕ, it is of interest to note that

$$\phi(x) = \phi(2x) + \phi(2x-1) \tag{3}$$

In general, it will be required that ϕ satisfies the **dilation equation**

$$\phi(x) = \sum_k c_k \phi(2x-k) \tag{4}$$

a property that is crucial to what follows. Up to a suitable normalisation factor, relation (4) will define ϕ. The normalisation $\int \phi(x)\,dx = 1$ is chosen for the current considerations and this imposes the condition $\sum_k c_k = 2$. Function ϕ is known as the **scaling function.**

If coefficients c_k are zero, except possibly for some k between 0 and N, it can be shown [1] that the scaling function will be zero for points outside the interval [0,N]. (Except for the box function, it will normally be required that $\phi(0) = \phi(N) = 0$.)

From (4), the Fourier transform of ϕ is given by

$$\hat{\phi}(\omega) = \sum_k c_k \int \phi(2x-k)\exp(i\omega x)\,dx \tag{5}$$

and, on setting $y = 2x-k$,

$$\hat{\phi}(\omega) = \int \phi(y)\exp\left(\frac{i\omega y}{2}\right)dy \left(\frac{1}{2}\sum_k c_k \exp\left(\frac{ik\omega}{2}\right)\right) \tag{6}$$

Define

$$P(\omega) = \frac{1}{2}\sum_k c_k \exp(ik\omega) \tag{7}$$

then

$$\hat{\phi}(\omega) = \hat{\phi}\left(\frac{\omega}{2}\right)P\left(\frac{\omega}{2}\right) \tag{8}$$

from which

$$\hat{\phi}(\omega) = \prod_{j=1}^{N} P\left(\frac{\omega}{2^j}\right)\hat{\phi}\left(\frac{\omega}{2^N}\right) \tag{9}$$

In the limit $N \to \infty$, this implies

$$\hat{\phi} = \prod_{j=1}^{\infty} P\left(\frac{\omega}{2^j}\right) \tag{10}$$

since $\hat{\phi}(0) = 1$.

Condition A P has a zero of order p at $\omega = \pi$.

For m=0,1,...,p-1, this implies that

$$\sum_k (-1)^k k^m c_k = 0 \tag{11}$$

If condition A holds and f is a smooth function, coefficients a_k can be found [6] such that

$$\| f - \sum_k a_k \phi(2^J x - k) \| \le C 2^{-Jp} \| f^{(p)} \| \tag{12}$$

for some constant C. Such a result justifies the use of (2) as an approximation to f.

3 CALCULATION OF THE SCALING FUNCTION

Consider relation (4) at integer points from 1 to N-1 ($c_k = 0$ for $k > N$ and $k < 0$). This provides the system

$$\begin{pmatrix} \phi(1) \\ \phi(2) \\ \vdots \\ \phi(N-1) \end{pmatrix} = T \begin{pmatrix} \phi(1) \\ \phi(2) \\ \vdots \\ \phi(N-1) \end{pmatrix} \tag{13}$$

where T has a maximum eigenvalue of 1. The power method can be used to find the corresponding eigenvector and, up to a scaling factor, the components of this eigenvector will provide values for $\phi(1), \ldots, \phi(N-1)$. Relation (4) can be used to find intermediate values and, after sufficient of these have been calculated, the normalisation $\int \phi(x)\,dx = 1$ imposed by use of a suitable quadrature procedure.

Condition O

$$|P(\omega)|^2 + |P(\omega+\pi)|^2 = 1 \tag{14}$$

or

$$\sum_k c_k c_{k-2m} = 2\delta_{0m} \tag{15}$$

For fixed J, this implies that the ϕ_{iJ} are orthonormal.

Conditions O and A are desirable properties and will be satisfied [1] if P has the form

$$P(\omega) = \left(\frac{1+\exp(i\omega)}{2}\right)^L Q(\omega) \tag{16}$$

where

$$|Q(\omega)|^2 = \sum_{j=0}^{L-1} \binom{L-1+j}{j} (\sin^2\frac{\omega}{2})^j + (\sin^2\frac{\omega}{2})^L R(\frac{\cos\omega}{2}) \tag{17}$$

and R is a real odd function.

The box scaling function corresponds to $L=1$ and $R=0$ so that $P(\omega)=\frac{1+\exp(i\omega)}{2}$. Although this scaling function is discontinuous, others of the same class become smoother as L increases (at least C^0 for L=2 , C^1 for L=4 and C^2 for L=7).

4 THE WAVELET FUNCTION

Consider the approximations $A_{J+1}f$ and A_Jf. It can be shown [6] that

$$A_{J+1}f = A_Jf + D_Jf \tag{18}$$

where

$$D_Jf = \sum_i d_i^J \psi_{iJ} \tag{19}$$

and $\psi_{ij} = 2^{\frac{j}{2}}\psi(2^jx-i)$ for a suitable **wavelet function** ψ. In the case of the box scale function, the corresponding ψ is given by

$$\psi(x) = \begin{matrix} 1 & x\in(0,\frac{1}{2}) \\ -1 & x\in(\frac{1}{2},1) \\ 0 & otherwise \end{matrix} \tag{20}$$

and is known as the Haar wavelet. It will be noted that

$$\psi(x) = \phi(2x) - \phi(2x-1) \tag{21}$$

In general, the wavelet corresponding to the scale function ϕ is given by

$$\psi(x) = \sum_k (-1)^k c_{1-k}\phi(2x-k) \tag{22}$$

This construction ensures that $\psi(x)$ is orthogonal to $\phi(x-m)$ for $m \in Z$. Furthermore, condition O implies that ψ_{ij} ($i \in Z$) forms an orthonormal basis for the $D_j f$. Repeating the reduction of equation (18)

$$A_{J+1}f = D_J f + D_{J-1}f + D_{J-2}f + D_{J-3}f + \ldots + D_{J-K}f + A_{J-K}f \tag{23}$$

and taking limits in J and K,

$$f = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} d_i^j \ \psi_{ij} \tag{24}$$

It can be shown [1] that ψ_{ij} ($i,j \in Z$) form a orthonormal basis for functions in $L^2(R)$ (the space of Lebesgue square integrable functions).

If condition A holds, wavelet functions have the following important properties:

(1) $\int x^m \psi(x)\, dx = 0$ for m=0 to p-1.

(2) For a C^∞ function f, $d_i^j \le C 2^{-j(p-\frac{1}{2})}$.

Daubechies [1] has studied scaling functions that are defined by (16) and has calculated the coefficients c_k for R=0 and L from 1 to 10. For example, the box function corresponds to L=1 and has $c_0 = c_1 = 1$. For L=2, $c_0 = (1+\sqrt{3})/4$, $c_1 = (3+\sqrt{3})/4$, $c_2 = (3-\sqrt{3})/4$, $c_3 = (1-\sqrt{3})/4$ and $c_k = 0$ otherwise. Figures 2 ,3 and 4 show the scaling functions, wavelet functions and their corresponding Fourier transforms for the cases where L=1, 2 and 7. From these examples, it can be seen that the scaling function has the nature of a low pass filter and the wavelet function the nature of a band pass filter.

5 WAVELET ANALYSIS

For most applications, the function of interest is only known in terms of a finite number of samples. In order to perform a wavelet analysis, these samples must be turned into an initial approximation $A_J f$. A wavelet analysis decomposes $A_J f$ into its constituent projections ($D_{J-1}f$, $D_{J-2}f$, $D_{J-3}f$, ...). Equivalently, the coefficients $\{a_i^J\}$ of $A_J f$ are decomposed into the coefficients ($\{d_i^{J-1}\}$, $\{d_i^{J-2}\}$, ...) of the constituent projections. A major advantage of wavelet analysis is the existence of fast algorithms for performing this reduction. The present section describes these algorithms and their application. In addition, the interpretation of a wavelet analysis is discussed.

Consider the decomposition

$$A_{J+1}f = A_J f + D_J f \tag{25}$$

or, in terms of the basis functions,

$$\sum_i a_i^{J+1}\phi_{i,J+1} = \sum_i a_i^{J}\phi_{i,J} + \sum_i d_i^{J}\psi_{i,J} \tag{26}$$

where $a_i^J = \langle f, \phi_{i,J}\rangle$ and $d_i^J = \langle f, \psi_{i,J}\rangle$. (The inner product $\langle f, g\rangle$ is defined by $\int_R f(u)g(u)\,du$.)

Given a_i^{J+1}, what are a_i^J and d_i^J? From the definition of $\phi_{i,J}$ and relation (4),

$$\begin{aligned} a_i^J &= \langle f, \phi_{i,J}\rangle \\ &= 2^{\frac{J}{2}}\int_R f(u)\phi(2^J u - i)\,du \\ &= 2^{\frac{J}{2}}\sum_k c_k \int_R f(u)\phi(2^{J+1}u - 2i - k)\,du \\ &= 2^{\frac{J}{2}}\sum_\kappa c_{\kappa-2i}\int_R f(u)\phi(2^{J+1}u - \kappa)\,du \end{aligned} \tag{27}$$

if $\kappa = 2i + k$. Since $a_\kappa^{J+1} = \langle f, \phi_{\kappa,J+1}\rangle = 2^{\frac{J+1}{2}}\int_R f(u)\phi(2^{J+1}u - \kappa)\,du$, it follows that

$$a_i^J = \frac{1}{\sqrt{2}}\sum_\kappa c_{\kappa-2i}\, a_\kappa^{J+1} \tag{28}$$

Consequently, to form the coefficients $\{a_i^J\}$, convolve coefficients $\{a_i^{J+1}\}$ with the filter coefficients $\{\frac{c_{-\kappa}}{\sqrt{2}}\}$ and keep alternate samples (a low pass filtering). In a similar fashion

$$d_i^J = \frac{1}{\sqrt{2}}\sum_\kappa (-1)^\kappa c_{1-\kappa+2i}\, a_\kappa^{J+1} \tag{29}$$

Consequently, to form the coefficients $\{d_i^J\}$, convolve coefficients $\{a_i^{J+1}\}$ with the filter coefficients $\{\frac{(-1)^\kappa c_{-1+k}}{\sqrt{2}}\}$ and keep alternate samples (a band pass filtering).

It is also possible to reconstruct the coefficients $\{a_i^{J+1}\}$ from the coefficients $\{a_i^J\}$ and $\{d_i^J\}$. From (28), it follows that

$$<\phi_{k,J+1}, \phi_{i,J}> = \frac{c_{k-2i}}{\sqrt{2}} \tag{30}$$

and, from (29),

$$<\phi_{k,J+1}, \psi_{i,J}> = \frac{(-1)^k c_{1-k+2i}}{\sqrt{2}} \tag{31}$$

Consequently, on taking the inner product of $\phi_{i,J+1}$ with (26),

$$a_i^{J+1} = \frac{1}{\sqrt{2}}\sum_j a_j^J c_{i-2j} + \frac{1}{\sqrt{2}}\sum_j d_j^J (-1)^i c_{1-i+2j} \tag{32}$$

In effect, coefficients $\{a_i^{J+1}\}$ are formed from coefficients $\{a_i^J\}$ and $\{d_i^J\}$ by adding the results of two convolutions. The first convolution involves the coefficients $\{a_i^J\}$ and $\{\frac{c_k}{\sqrt{2}}\}$ and the second involves the coefficients $\{d_i^J\}$ and $\{\frac{(-1)^k c_{1-k}}{\sqrt{2}}\}$.

In order to produce a practical implementation of the above procedures, it is necessary to work with a finite amount of data. Unfortunately, the decomposition of $\underline{a}^{J+1} = (a_0^{J+1}, a_1^{J+1}, ..., a_{2S-1}^{J+1})$ into $\underline{a}^J = (a_0^J, a_1^J, ..., a_{S-1}^J)$ and $\underline{d}^J = (d_0^J, d_1^J, ..., d_{S-1}^J)$ requires elements that are not contained in $\underline{a}^{J+1}$. If these elements are unknown, one possible procedure is to zero pad the known elements. It should be noted, however, that this approach will lead to an increasing number of incorrect coefficients as the decomposition proceeds. After several steps of decomposition, many of the highest N, and lowest N, components of the $\underline{a}^J$ and $\underline{d}^J$ will be contaminated. An alternative approach is to assume that the initial data represents one period of a periodic function. It should be noted, however, that a periodic function is not square integrable and so a complete decomposition into wavelets is impossible. In this case, the decomposition will be meaningless beyond K steps for initial data consisting of 2^K samples.

A further problem in the analysis of discrete data is the derivation of the initial coefficients $\{a_i^J\}$. These are calculated according to

$$\begin{aligned} a_i^J &= \langle f, \phi_{iJ} \rangle \\ &= 2^{\frac{J}{2}} \int_R f(u)\,\phi(2^J u - i)\, du \end{aligned} \tag{33}$$

In effect, the coefficients are obtained by sampling the function after passing it through a low pass filter that corresponds to a suitably scaled ϕ. Consider the samples $\{f_i^J\}$ taken from f at intervals of 2^{-J}. The coefficients $\{a_i^J\}$ can be approximated according to

$$a_i^J \approx 2^{-\frac{J}{2}} \sum_{k=i}^{i+N} f(2^{-J}k)\,\phi(k-i) \tag{34}$$

or, in a more convenient form,

$$a_i^J \approx 2^{-\frac{J}{2}} \sum_{k=0}^{N} \phi_k\, f_{k+i}^J \tag{35}$$

where $\phi_k = \phi(k)$. This is an approximate implementation of the filtration and sampling processes. Appendix A contains some FORTRAN subroutines for implementing (35) in the case of periodic data that has been sampled at unit intervals (initial J zero).

The wavelet decomposition is economical in terms of both storage and computational requirement. Firstly, the decomposition algorithm only requires $O(M)$ operations if the original data set has M elements. Secondly, the data storage can be accomplished in the same area as the original data since $\underline{a}^J$ becomes $(\underline{a}^{J-1}, \underline{d}^{J-1})$ which then becomes $(\underline{a}^{J-2}, \underline{d}^{J-2}, \underline{d}^{J-1})$ and so on. Appendix A contains some FORTRAN subroutines for the wavelet decomposition (and reconstruction) of periodic data.

A wavelet decomposition, can be crudely regarded as a spectral analysis with frequency space divided into bands of width proportional to their centre frequency (constant Q filtration). Furthermore, the analysis exhibits some localisation in time (the domain of f). Referring to Figure 5, the basis function corresponding to a given wavelet coefficient will make its major contribution over the region that is delimited by the box surrounding the coefficient. Crudely, the square moduli of the wavelet coefficients yield the distribution of energy in terms of time and frequency. Referring to Figure 6, the various projections ($D_J f$, $D_{J-1} f$, ...) represent the original function after it has passed through the filters corresponding to the different bands. Figure 7 shows the projections for a sampled chirp function and analysing Daubechies wavelet with L=7. As is to be expected, the projections migrate across the frequency plane as time advances. Figure 8 shows the projections for a cubed sine wave. It will be noted that the signal has been split into components that exhibit its two constituent frequencies.

Consider some sine wave data (Figure 9) and some noisy sine wave data (Figure 10). (Both data sets consist of 128 samples). Figures 11 and 12 show the respective wavelet expansion coefficients for the Daubechies wavelet with L=4. Starting at the bottom, the horizontal rows represent $\underline{d}^{-1}$, $\underline{d}^{-2}$, $\underline{d}^{-3}$, $\underline{d}^{-4}$, $\underline{d}^{-5}$ and $\underline{d}^{-6}$ (negative values indicated by the lighter shading). Quantities $\underline{a}^{-7}$ and $\underline{d}^{-7}$ are represented in order by the vertical columns. (The number in the top right hand box is the figure height in the coefficient scale). As is to be expected, the noise components are confined to the finer scales.

6 SIGNAL STRUCTURE

The wavelet coefficients $\{d_i^j\}$ can provide valuable information concerning the regularity of a signal [8], regularity described in terms of Lipschitz exponents. Function f is Lipschitz α at x_0 if there exists positive A and h_0 such that

$$|f(x) - P_n(x-x_0)| \leq A\,|x - x_0|^{\alpha} \tag{36}$$

for some polynomial P_n (order n) and $|x-x_0|<h_0$.

Function f is uniformly Lipschitz α over an interval if (36) holds for all pairs of points (x,x_0) in that interval. The following result [9] can be proved:

For a wavelet with greater than n vanishing moments, a function f is uniformly Lipschitz α if and only if there exists $K>0$ such that

$$|d_i^j| \leq K2^{-(\frac{2\alpha+1}{2})j} \tag{37}$$

for all j.

If the wavelet has compact support, it is obvious that the coefficients at finer scales will provide localised information concerning the Lipschitz exponents α of the function.

Although the above result does not extend directly to negative exponents, the behaviour of the wavelet coefficients can still provide a good indication of α. Figure 13 shows data sampled from a function that contains a pulse, a step and a continuous transition. The corresponding wavelet coefficients (Daubechies wavelet with L = 10) are shown in Figure 14. It will be noted that the coefficient maxima exhibit the expected behaviour (the features have exponents α with values -1, 0 and 1 respectively). This ability to provide information about transient features constitutes one of the major attractions of a wavelet approach.

Local results are more difficult to derive, but the following theorem has been proved by Jaffard [9]. If f is Lipschitz α at x_0,

$$|d_i^j| \leq C 2^{-(\frac{1+2\alpha}{2})j} (1 + |2^j x_0 - i|^{\alpha}) \tag{38}$$

Conversely, if equation (38) holds and f is uniformly C^{β} for a positive number β, there exists a polynomial P (depending on x_0) of degree less than α such that

$$|f(x) - P(x - x_0)| \leq C\, |x - x_0|^{\alpha} \log\left(\frac{2}{|x - x_0|}\right) \tag{39}$$

and this result is optimal.

7 CONCLUSIONS

The present report has described discrete wavelet analysis through an approach that was pioneered by Daubechies [1,6]. This approach has been developed into a computer software package (Appendix B) that can be used to analyse sampled data. The report has included many examples that were produced using this package and these amply demonstrate the potential of a wavelet approach to signal analysis. A major advantage of the approach is its ability to provide a degree of time localisation and this, together with the existence of very fast algorithms, makes it a powerful tool for analysing signals with transient features. In particular, wavelets have found an important application in the analysis of speech [7]. It should be noted, however, that the approach only provides spectral information in terms of frequency bins that have constant Q and so may not be suitable for applications requiring a high degree of frequency resolution.

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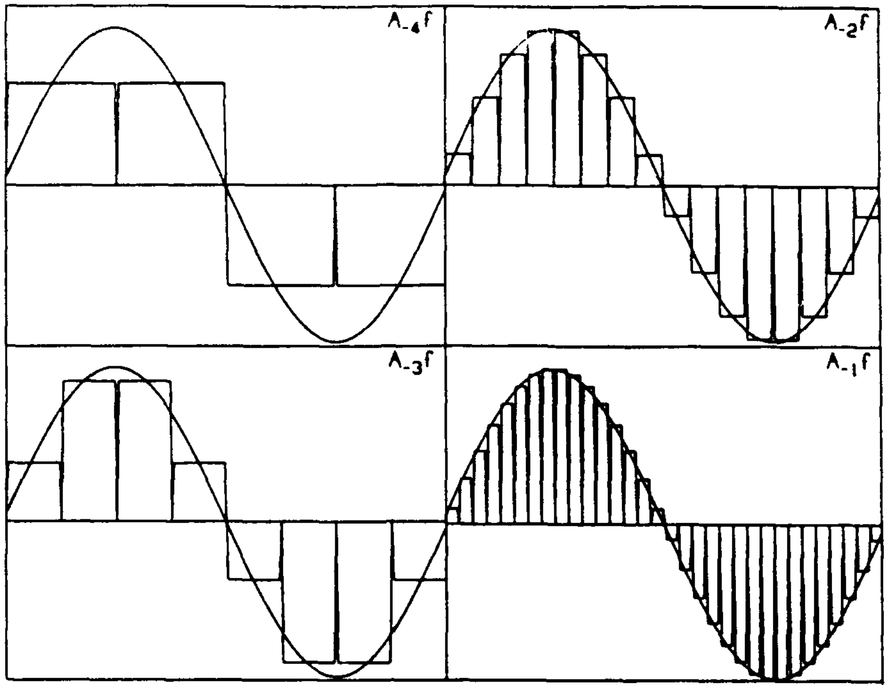

Figure 1 Box function approximations.

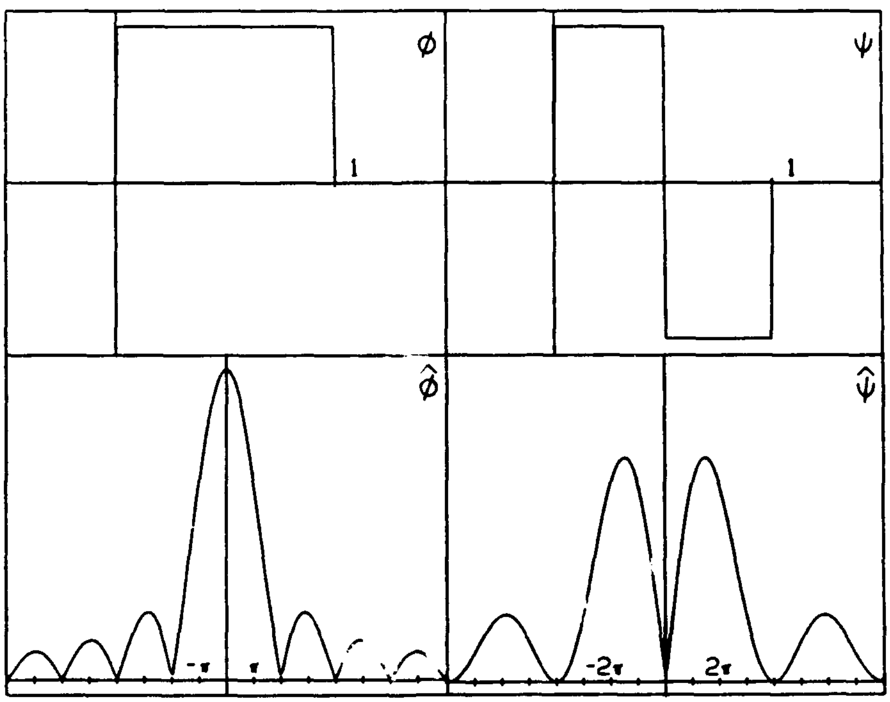

Figure 2 Scale and wavelet functions for L = 1.

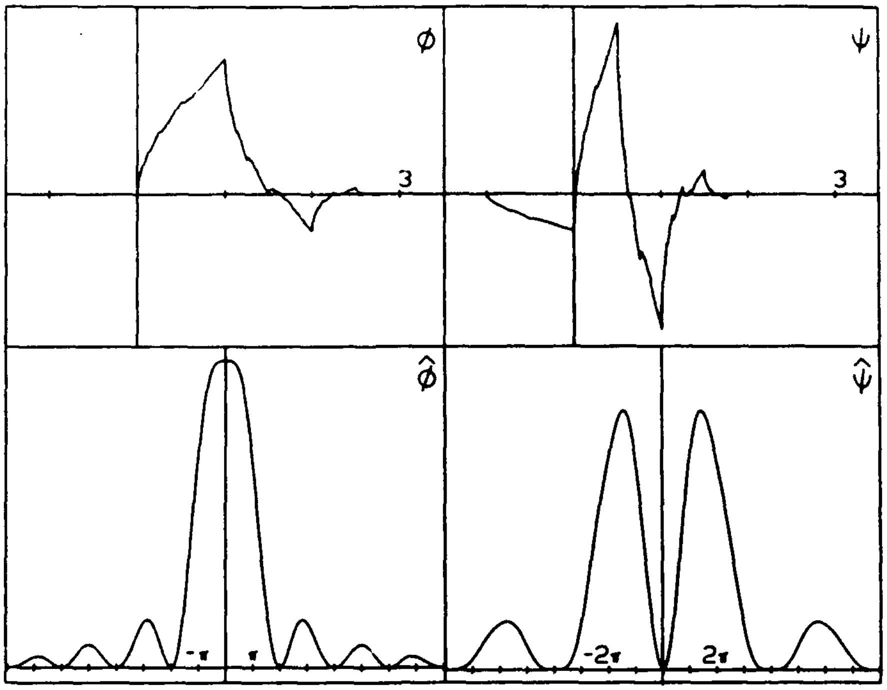

Figure 3 Scale and wavelet functions for L = 2.

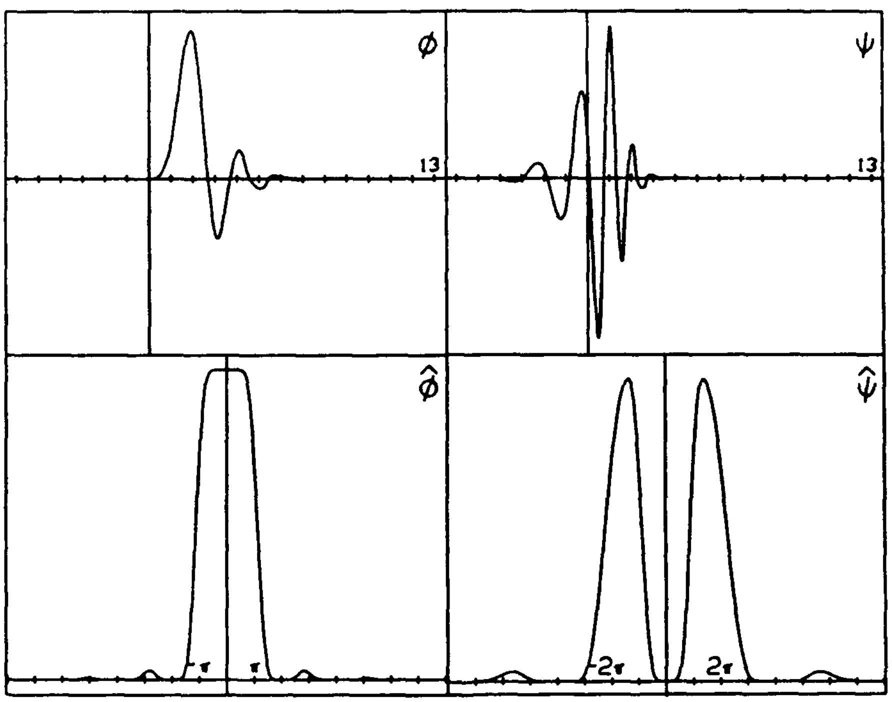

Figure 4 Scale and wavelet functions for L = 7.

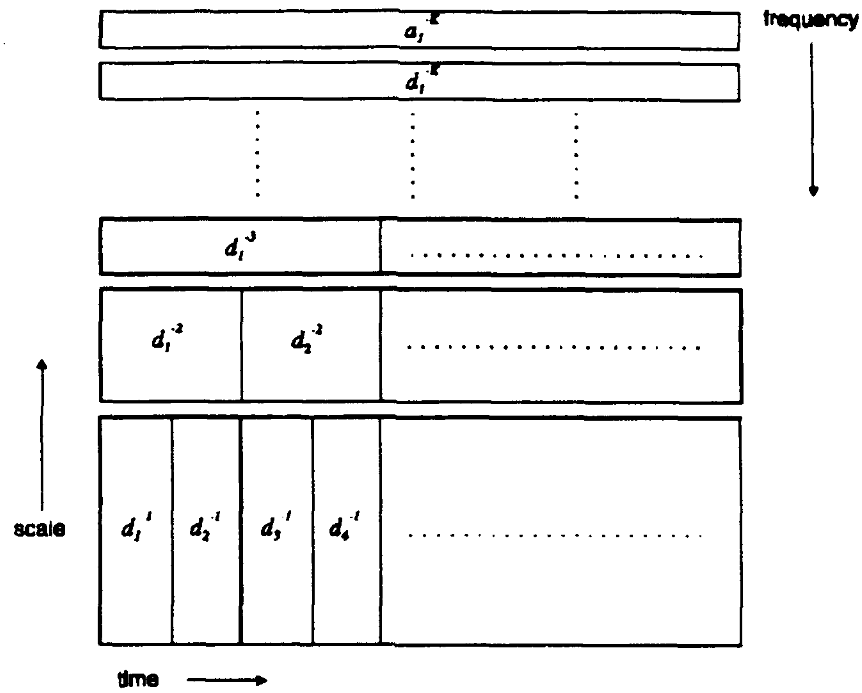

Figure 5 Influence of wavelet coefficients in terms of frequency and time.

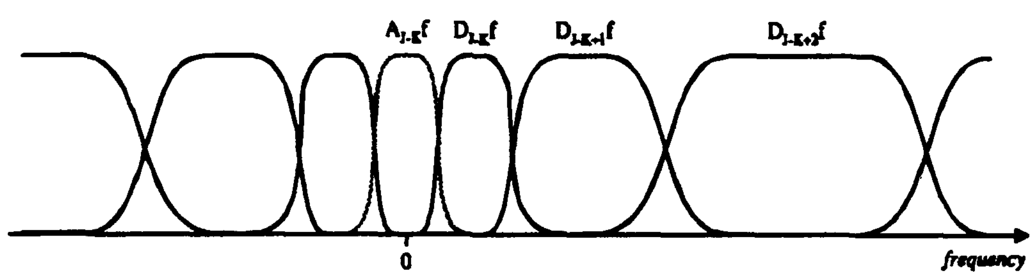

Figure 6 Frequency range of projections.

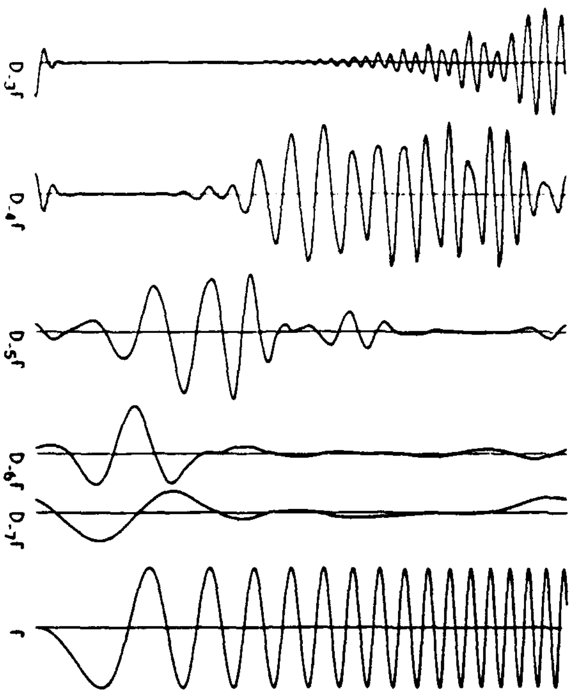

Figure 7 Decomposition of a chirp.

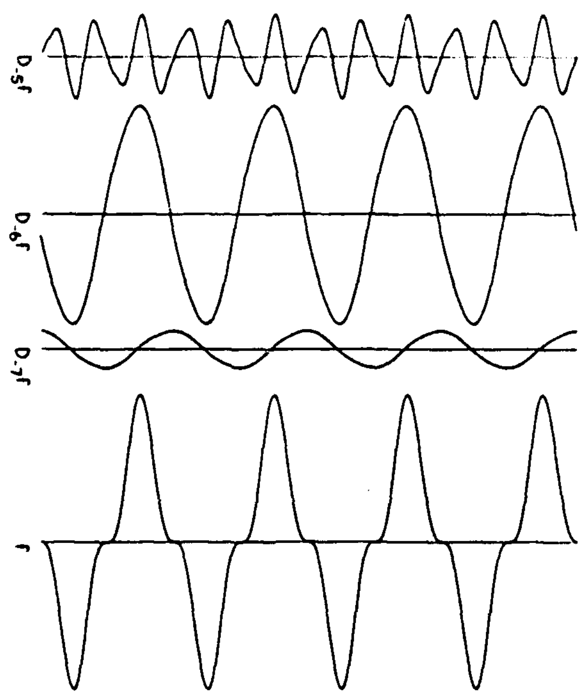

Figure 8 Decomposition of a cubed sine wave.

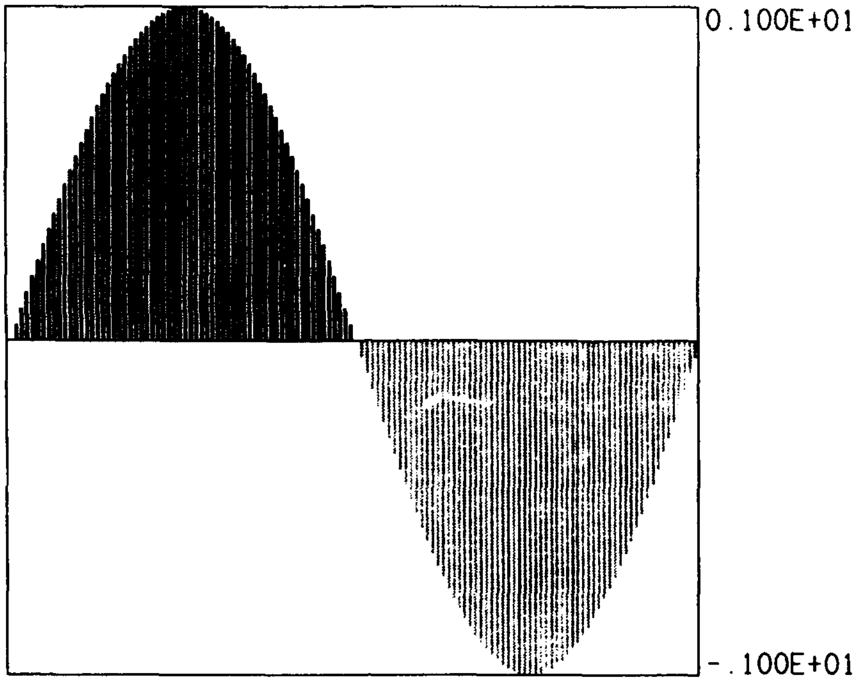

Figure 9 Sine wave.

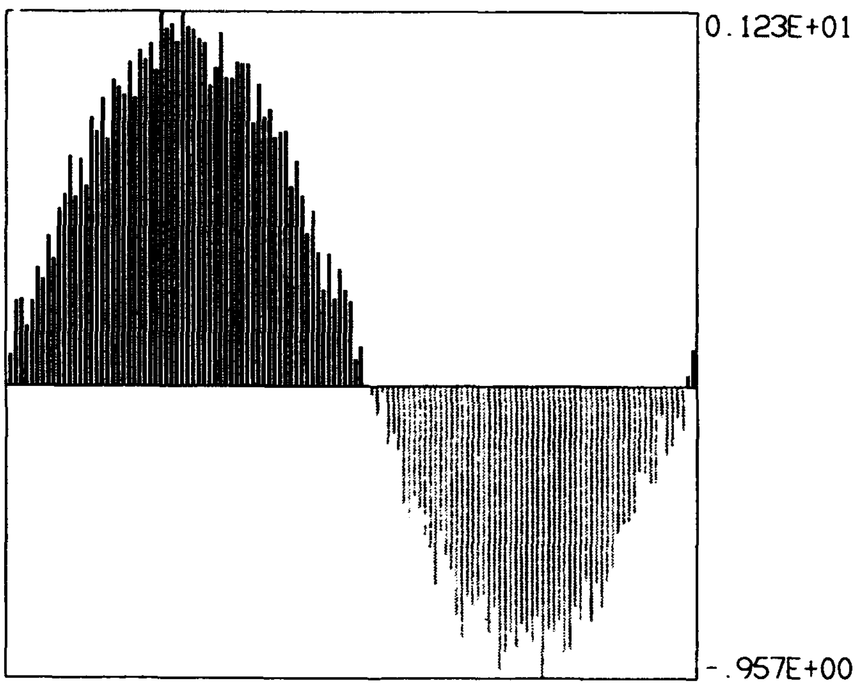

Figure 10 Noisy sine wave.

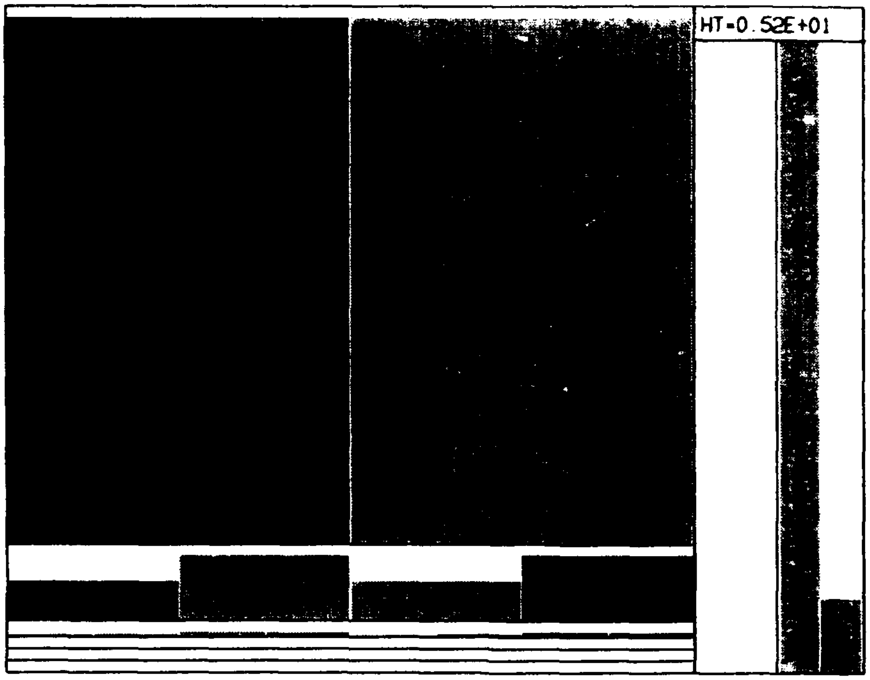

Figure 11 Wavelet coefficients for sine wave.

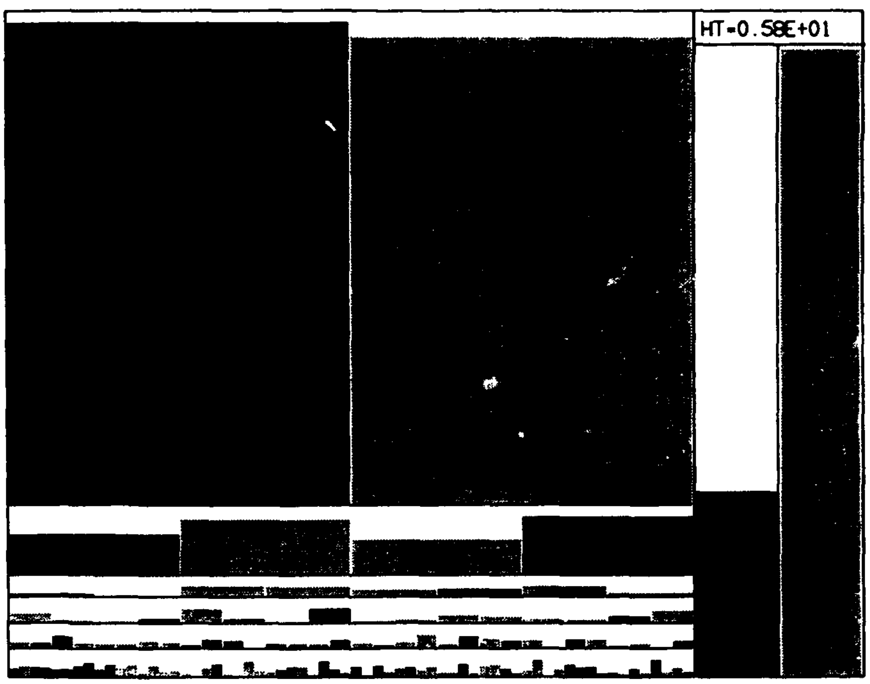

Figure 12 Wavelet coefficients for noisy sine wave.

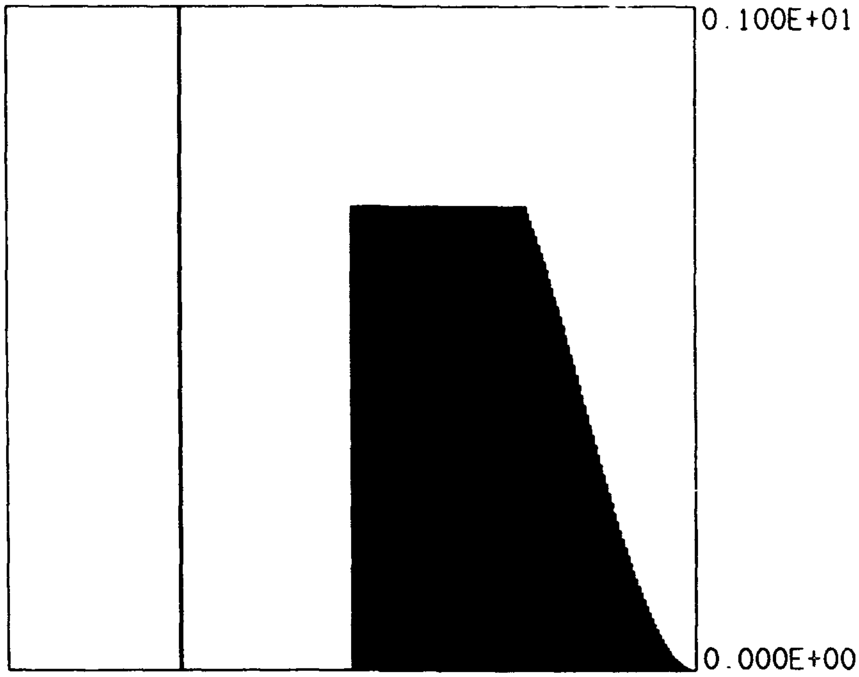

Figure 13 Data from function with singular behaviour.

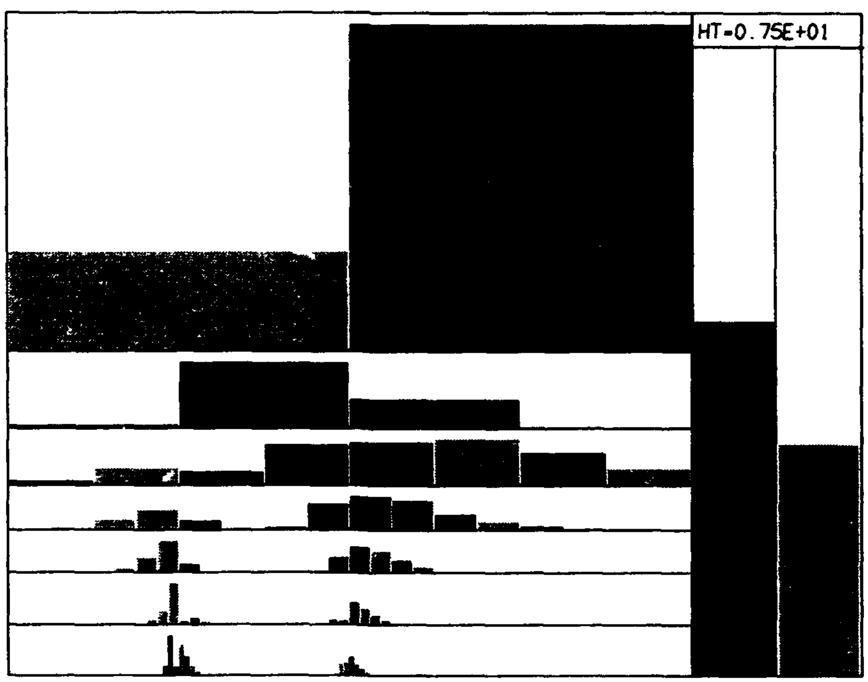

Figure 14 Wavelet coefficients for function with singular behaviour.

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APPENDIX A COMPUTER SUBROUTINES

The following FORTRAN subroutines are the minimum that is required to implement a wavelet analysis. Coefficients c_0 to c_{N-1} (all other coefficients zero) should be placed in the array elements c(1) to c(N). Subroutine **scale** places the values of scale function ϕ at points 0 to N-1 into the array elements phi(1) to phi(N). The function of interest is sampled at M points ($M = 2^K$) with corresponding values f(1) to f(M). Subroutine **fit** will estimate the filtered samples that are used in the initial approximation $A_0 f$ and place them in the array elements a(1) to a(M). The wavelet analysis is performed by subroutine **decomp** which replaces a(1) to a(M) with the wavelet coefficients ($\underline{a}^{-K}$, $\underline{d}^{-K}$, $\underline{d}^{-K+1}$,....., $\underline{d}^{-1}$). Finally, subroutine **recons** can be used to undo the above analysis to a level where there are ML scaling function basis terms.

```
**********************************************************
*** Values of scaling function phi at points 0 to N-1 ***
**********************************************************
      subroutine scale(c,N,phi)
      dimension c(N),phi(N),a(51,51),e(51)
      n1=N-1
      do 11 i=2,n1
      do 11 j=2,n1
      cc=0.0
      ij=2*(i-1)-j+1
      if(ij.ge.0.and.ij.lt.N) cc=c(ij+1)
11    a(i-1,j-1)=cc
      em=0.0000001
      n2=n1-1
      n3=51
      call maxev(a,e,em,n2,n3)
      phi(1)=0.0
      phi(N)=0.0
      do 22 i=2,n1
22    phi(i)=e(i-1)
      ph=0.0
      do 33 i=1,N
33    ph=ph+phi (i)
      do 44 i=1,N
44    phi (i)=phi(i)/ph
      return
      end
************************************************************
*** eigenvector corresponding to the maximum eigenvalue ***
************************************************************
      subroutine maxev(a,e,emax,n,m)
      dimension a(m,n),e(n),v(200)
      if(n.eq.1) then
      e(1)=1.
      emax=a(1,1)
      return
      endif
      error=emax
      do 1 i=1,n
1     e(i)=1.0
99    do 4 i=1,n
      v(i)=0.0
      do 4 j=1,n
4     v(i)=v(i)+a(i,j)*e(j)
      eold=emax
      do 2 i=1,n
2     if(abs(v(i)).gt.abs(emax)) emax=v(i)
      do 3 i=1,n
3     e(i)=v(i)/emax
      if(abs((emax-eold)/eold).lt.error) return
      go to 99
```

```
      end
***********************************************************
*** production of filtered samples from function samples ***
***********************************************************
      subroutine fit(a,M,f,phi,N)
      real*4 a(m),f(ns),phi(N)
      do 1 i=1,ns
      a(i)=0.0
      do 1 j=1,N
      ix=1+j-2
      ix=mod(ix,M)
1     a(i)=a(i)+phi(j)*f(ix+1)
      return
      end
*************************************
*** convolution of arrays x and g ***
*************************************
      real*4 function conv(g,n,na,x,m,1)
      real*4 g(n),x(m)
      conv=0.0
      do 1 i=1,n
      ix=1-i-na
      if(ix.lt.0) ix=ix+m
      ix=mod(ix,m)
1     conv=conv+g(i)*x(ix+1)
      return
      end
*************************************************
*** wavelet coefficients from filtered samples ***
*************************************************
      subroutine decomp(a,M,c,N)
      real*4 a(M),c(n),g1(100),g2(100),ft(1024),dt(1024)
      n1=-N+1
      n2=-1
      sqr2=sqrt(2.)
      do 5 i=1,N
      g1(i)=c(N-i+1)/sqr2
5     g2(i)=c(i)*real((-1)**i)/sqr2
      nt=M
1     nn=nt
      nt=nt/2
      do 2 j=1,nt
      k=j*2-1
2     ft(j)=conv(g1,N,n1,a,nn,k)
      do 3 j=1,nt
      k=j*2-1
3     ft(j)conv(g2,N,n2,a,nn,k)
      do 4 j=1,nt
      a(j+nt)=dt(j)
4     a(j)=ft(j)
      if(nt.lt.2) then
      return
      else
      go to 1
      endif
      end
**********************************************************
*** reconstitution of samples from wavelet coefficients ***
**********************************************************
      subroutine recons (f,ML,c,N)
      real*4 f(M),c(n),g1(100),g2(100),ft(1024),dt(1024)
      n1=0
      n2=-N+2
      sqr2=sqrt(2.)
      do 5 i=1,N
      g1(i)=c(i)/sqr2
```

```
5     g2(i)=real((-1)**(i+1))*c(N-i+1)/sqr2
      nt=1
1     nn=nt
      do 2 j=1,nn
      ft(2*j-1)=f(j)
      ft(2*j)=0.0
      dt(2*j-1)=f(nn+j)
2     dt(2*j)=0.0
      nt=nt*2
      do 3 j=1,nt
3     f(j)=conv(g1,N,n1,ft,nt,j)+conv(g2,N,n2,dt,nt,j)
      if(nt.ge.ML) then
      return
      else
      go to 1
      endif
      end
```

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APPENDIX B SOFTWARE

The WV wavelet software package runs on a DOS PC and has the following facilities:

1) To choose a Daubechies wavelet and to display the wavelet function, the scaling function and their respective Fourier transforms.
2) To enter and filter data.
3) To display data.
4) To display a frequency analysis of data (the amplitude of the discrete Fourier transform).
5) To change to a logarithmic data scale.
6) To change to an exponential data scale (reverses 5).
7) To calculate the wavelet coefficients corresponding to user supplied data. The coefficients are displayed in the format of Figures 11, 12 and 14. For a data set of initial size 2^K (data sampled at unit intervals), the rows from bottom to top represent the coefficients of projections $D_{-1}f, D_{-2}f, .., D_{1-K}f$. The first column represents the coefficients of $A_{-K}f$ and the second column represents the coefficients of $D_{-K}f$. The top right hand box shows the display height in the coefficient scale.
8) To display the projections $A_{-K}f, D_{-K}f, D_{-K+1}f, ..., D_{-1}f$ (only those projections above a certain threshold will be displayed).
9) To output a graphics screen as a file of postscript instructions.
10) To exit the package.

It should be noted that a graphics screen is exited by pressing the enter key and that the software can accept at most 1024 data samples (free format).

The computer disc contains the WV software and some sample data.

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